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An evolutionary game model of financial markets with heterogeneous players

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Abstract

Three types of market traders, including momentum traders, contrarian traders and fundamentalists, are introduced to an evolutionary game model as market players, and their payoff structures are given. Based on a discrete replicator equation, a dynamic system is defined, and then its evolutionarily stable states are presented, which correspond to different market price evolving processes, including the stationary price fluctuation around the fundamental value, the increasing (decreasing) price bubble and the stationary, fluctuating positive (negative) price bubble.

Keywords: Evolutionary game; heterogeneous trader; price dynamics; price bubble.

1. Introduction

In the last decade, the literature on modeling financial markets by incorporating heterogeneous agents and bounded rationality has increased[1-8]. These models share two fundamental characteristics. The first is that the heterogeneous agents have different expectations about the future price evolution and have different forms of the asset demand function. Therefore they result in different market performance (measured by the expected profit or realized profit). The second is that the heterogeneous agents can adapt their beliefs over time by choosing from different expectation patterns and changing their market strategies based upon their past performance, and heterogeneity, learning, adaptation and evolution can be incorporated into this type of adaptive belief system [9-10]. Obviously, these fundamental characteristics make the modeled financial market

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analogous to the ecological system. This means that we can introduce evolutionary game theory to study the financial market. Evolutionary game theory focuses on the dynamics of strategy change more than the properties of strategy equilibria and avoids strong assumptions of rationality to admit bounded rationality[11-12]. The successful application of this theory has brought further insights to financial systems[13].

Therefore, in this paper, we will adopt evolutionary game theory to model an asset market with classic heterogeneous traders, composed of fundamentalists, momentum traders and contrarian traders. The momentum and contrarian strategies are two basic technical strategies in financial market[14]. Here we suppose that momentum traders hold trend-following expectations and contrarian strategy traders conceive trend-reversal expectations on the asset price. In this paper, we present different evolutionarily stable states of the evolutionary game and the corresponding market price dynamics, including the stationary price oscillation around fundamental value, the increasing (decreasing) price bubble and the stationary, fluctuating positive (negative) price bubble. Compared with the rational bubble theory[15], this model incorprates the stationary fluctuating positive (negative) price bubble to the increasing (decreasing) price bubble. In this sense, these results can be regarded as an extension to the rational bubble theory. Considering the bounded rationality of market traders, bubbles in our model can be termed as "boundedly rational bubbles". The remainder of this paper is organized as follows. Section 2 introduces the model. Section 3 contains the main theoretical analysis and results. Section 4 concludes.

2. The model

In this model, we suppose a population of three group players: the first group comprises momentum traders with a share of *x*, the second group comprises contrarian traders with a share of *y*, and the third group comprises fundamentalists with a share of 1-*x*-*y*. All the traders can shift from one group to another, and are matched randomly at each trading period.

When a trader meets another trader who is in the same group, they share homogeneous expectations and will have no real trade, leading to a zero-payoff. Thus, trade only happens between different groups. In particular, when a momentum trader meets a contrarian trader, they hold heterogeneous expectations and will trade with each other. We suppose that the excess demand of a momentum trader is $e_M = sgn(\Delta p_t)$, and the excess demand of a contrarian trader is $e_C = -sgn(\Delta p_t) = -e_M$, where $sgn(\pm) = \pm 1$ is a sign function. Then the payoff of a momentum trader is $e_M \Delta p_{t+1}$, and the payoff of a contrarian trader is $e_C \Delta p_{t+1}$. A fundamentalist's excess demand is $e_F = sgn(p_f - p_t)$; when he matches with a momentum trader or a contrarian trader, he can only achieve a trade if he and his counterpart have opposite excess demand , which means his payoff would be $e_M \Delta p_{t+1} H(-e_M e_F)$ or $e_C \Delta p_{t+1} H(-e_C e_F)$, where $H(+,-) = (0,1)$ is a Heaviside step function. In a similar way, we can obtain the payoff structure of all the three group traders as shown in Table 1, where "*M*" denotes momentum trader, "*C*" denotes contrarian trader, and "*F"* denotes fundamentalist.

Table 1. Payoff structure of the game with the momentum trader (M), contrarian trader (C) and fundamentalist(F).

	M (x)	R (y)	F (1-x-y)
M （x）	0	$e_M \Delta p_{t+1}$	$e_M \Delta p_{t+1} H(-e_M e_F)$
R (y)	$e_R \Delta p_{t+1}$	0	$e_R \Delta p_{t+1} H(-e_R e_F)$
F (1-x-y)	$-e_M \Delta p_{t+1} H(-e_M e_F)$	$-e_R \Delta p_{t+1} H(-e_R e_F)$	0

Supposing the price change is determined by the excess market demand, we take price dynamics as $\Delta p_{t+1} = \alpha[xe_M + ye_R + (1-x-y)e_F]$,which leads to

$$A_{MR} = e_M \Delta p_{t+1} = \alpha[x - y + (1-x-y)e_M e_F] = \begin{cases} \alpha(1-2y) & e_M e_F = 1 \\ \alpha(2x-1) & e_M e_F = -1 \end{cases} , \tag{1}$$

$$A_{MF} = e_M \Delta p_{t+1} H(-e_M e_F) = \begin{cases} 0 & e_M e_F = 1 \\ \alpha(2x-1) & e_M e_F = -1 \end{cases} , \tag{2}$$

$$A_{RF} = e_R \Delta p_{t+1} H(-e_R e_F) = \begin{cases} \alpha(2y-1) & e_M e_F = 1 \\ 0 & e_M e_F = -1 \end{cases} , \tag{3}$$

Thus, the expected payoff of a momentum trader is

$$\pi_M = yA_{MR} + (1-x-y)A_{MF} = \alpha y(1-2y)\cdot 1_{\{e_M e_F = 1\}} + \alpha(1-x)(2x-1)\cdot 1_{\{e_M e_F = -1\}} , \tag{4}$$

the expected payoff of a contrarian trader is

$$\pi_R = xA_{RM} + (1-x-y)A_{RF} = \alpha(y-1)(1-2y)\cdot 1_{\{e_M e_F = 1\}} - \alpha x(2x-1)\cdot 1_{\{e_M e_F = -1\}} , \tag{5}$$

and the expected payoff of a neutralist is

$$\pi_F = xA_{FM} + yA_{FR} = \alpha y(1-2y)\cdot 1_{\{e_M e_F = 1\}} - \alpha x(2x-1)\cdot 1_{\{e_M e_F = -1\}} . \tag{6}$$

Where A_{ij} denotes the payoff matrix as shown in Table 1.

Then the expected payoff of the total population is

$$\pi = x\pi_M + y\pi_R + (1-x-y)\pi_F = 0 . \tag{7}$$

The continuous replicator equations can be given in the following form

$$\begin{cases} \dot{x} = x(\pi_M - \pi) = x\pi_M \\ \dot{y} = y(\pi_M - \pi) = y\pi_M \end{cases} . \tag{8}$$

Considering that we adopt price difference in market dynamics, here we use their corresponding discrete form to describe the population dynamics.

$$\begin{cases} \Delta x_{t+1} \equiv x_{t+1} - x_t = x_t \pi_M \\ \Delta y_{t+1} \equiv y_{t+1} - y_t = y_t \pi_M \end{cases} \tag{9}$$

Therefore, we have the following two equations to describe the dynamic system:

$$\begin{cases} \Delta x_{t+1} = x_t[\alpha y_t(1-2y_t)\cdot 1_{\{e_M e_F = 1\}} + \alpha(1-x_t)(2x_t-1)\cdot 1_{\{e_M e_F = -1\}}] \\ \Delta y_{t+1} = y_t[\alpha(y_t-1)(1-2y_t)\cdot 1_{\{e_M e_F = 1\}} - \alpha x_t(2x_t-1)\cdot 1_{\{e_M e_F = -1\}}] \end{cases} \tag{10}$$

The market price dynamics is described as

$$\Delta p_{t+1} = \alpha[(x_t - y_t)e_M + (1-x_t-y_t)e_F] \tag{11}$$

3. Equilibrium and evolutionarily stable state

First, without loss of the principle of stability analysis, we can divide the system evolution into three complete situations by the value of $e_M e_F$. The three situations are listed as follows:

(1) There exists time t', for every $t \geq t'$, $e_M e_F = 1$;

(2) There exists time t', for every $t \geq t'$, $e_M e_F = -1$;

(3) There exists time t', for every $t_0 \geq t'$, there exists time $t_1 \geq t_0$, $(e_M e_F)_{t_0}(e_M e_F)_{t_1} = -1, e_M e_F = -1$.

We do not include the situation of $e_M e_F = 0$ because we can always give the system minimal perturbation to make $e_M e_F \neq 0$, and this kind of minimal perturbation will not change the final results of the system stability.

We then discuss system the stability under the three situations one by one.

3.1. Situation 1: $\exists t', s.t. \forall t \geq t', e_M e_F = 1$

It can be proved that this situation can be excluded from the real world. The most straight-forward explanation for this argument is that $e_F = sgn(p_f - p_t)$ will eventually go to -1 (or +1) when $e_M = sgn(\Delta p_t)$ keeps taking 1 (or -1).

3.2. Situation 2: $\exists t', s.t. \forall t \geq t', e_M e_F = -1$

Just from the analysis of Situation 1, we can see that this situation can appear in the real world. Therefore, we can study the evolution of the dynamical system (1) after time t'. In this situation, the dynamic replicator equations can be reduced to

$$\begin{cases} \Delta x_{t+1} = \alpha x_t (1 - x_t)(2x_t - 1) \\ \Delta y_{t+1} = \alpha x_t y_t (1 - 2x_t) \end{cases} \tag{12}$$

The price dynamics is reduced to $\Delta p_{t+1} = \alpha(2x_t - 1)e_M$.

Solving equations (12) we can easily obtain the equilibria of the dynamic system: $(1,0),(0.5, m_1)$ and $(0, m_2)$, where $0 \leq m_1 \leq 0.5, 0 \leq m_2 \leq 1$.

Applying stability analysis to all the equilibria, we can obtain the evolutionary behaviors of system (11) when $e_M e_F = -1 (\forall t)$ (see Fig.1), where the equilibria $(1,0)$ and $(0, m_2)(0 \leq m_2 \leq 1)$ are evolutionarily stable states .

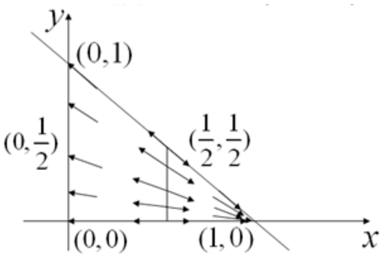
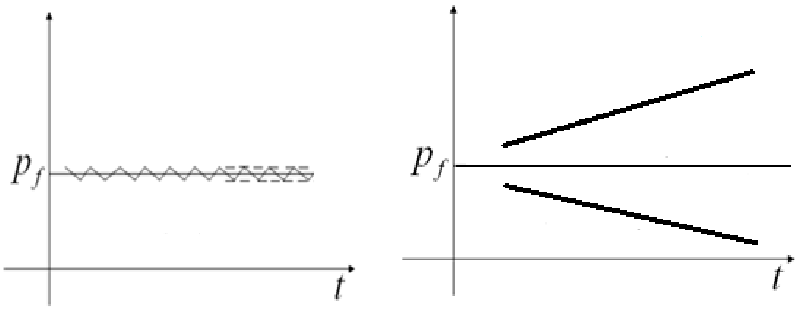


Fig.1. The equilibria $(1,0)$ and $(0, m_2)(0 \leq m_2 \leq 1)$ are evolutionarily stable, while $(0.5, m_1)(0 \leq m_1 \leq 1)$ are unstable. Every initial point in the area of $\{(x, y): x, y \geq 0, x + y \leq 1\}$ will converge to the steady state solution $(1,0)$ and $(0, m_2)(0 \leq m_2 \leq 1)$.

It is easy to obtain the market price dynamics in different evolutionarily stable states. For state $(0, m_2)(0 \le m_2 \le 1)$, which corresponds to the coexistence of momentum traders and fundamentalists, the price dynamics is a regular fluctuation around the fundamental value (see Fig.2.a). For the state $(1, 0)$, which corresponds to the mono-existence of momentum traders, the price dynamics is a persistently positive or negative bubble(see Fig.2.b).



(a) (b)

Fig. 2. (a) The price dynamics of the stable states $(0, m_2)(0 \le m_2 \le 1)$;(b) The price dynamics of the stable state of $(1, 0)$. The upward line corresponds to the positive initial value of e_M , and the downward line corresponds to the negative initial value of e_M .

3.3. Situation 3: $\exists t', \forall t_0 \ge t', \exists t_1 > t_0, s.t.(e_M e_F)_{t_0}(e_M e_F)_{t_1} = -1$

In this situation, there exists a switch in the sign of the term $e_M e_F$. Supposing that t_1 is the first period which satisfies $t_1 > t_0$ and $(e_M e_F)_{t_0}(e_M e_F)_{t_1} = -1$, we can prove that t_1 should be $t_0 + 1$. Exploring further, we can see that only the case[$(e_F)_{t+1} = (e_F)_t, (e_M)_{t+1} = -(e_M)_t$]can appear in the real world. In this case, there exists a unique, asymptotically stable equilibrium (0,1) (see Fig.3).

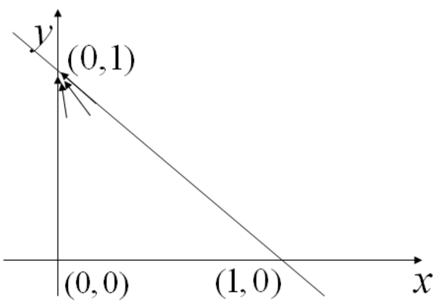


Fig.3. The equilibrium point $(0,1)$ is an evolutionarily stable state where $\forall t, (e_M)_t \cdot (e_M)_{t+1} = -1, (e_F)_t = (e_F)_{t+1}$.

The market price dynamics corresponding to the unique evolutionarily stable state with different initial prices are shown in Fig.4.

On the whole, the three situations give us a complete landscape of market traders' population evolution and price dynamic behavior. We have confirmed these conclusions with numerical simulations.

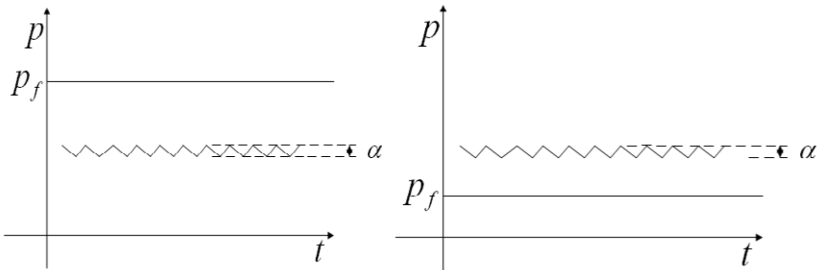


Fig.4. The price dynamics corresponding to the evolutionarily stable state (1,0)

4. Conclusion

In sum, the dynamic model presents three steady states with homogeneous traders, corresponding to sole fundamentalists, sole momentum traders and sole contrarian traders, respectively, and mixed states with fundamentalists matching contrarian traders. These steady states exhibit price dynamic processes, including fluctuations around the fundamental value, the increasing (decreasing) price bubble and the stationary, fluctuating positive (negative) price bubble. The highlights of this model lie in its straightforward dynamic mechanism of market evolution and neat results with strong economic meaning.

It is reasonable to argue whether the evolutionary scenarios in our model incarnate a real asset market process. At first glance, some of our evolutionarily stable states are far removed from real market behaviors; because they imply that the market participants eventually converge at a homogeneous trade strategy, and in these states the market will have no trade volume. In fact, not so as the general heterogeneous agent-based model, our model also cannot reproduce some common stylized facts in financial market, including excess volatility, skewness and excess kurtosis, fat tails, and volatility clustering[16].

Apparently, this disadvantage is mainly due to the simple assumptions of this model, which ignores some practical factors such as random shocks to fundamental value, noise in strategy shift, entry and exit of participants, liquidity demand. For instance, in real financial markets, there usually exist stubborn fundamentalists or trend traders, and passive liquidity traders, and their populations cannot be wiped out entirely. It is disappointing that these realistic factors are difficult to include in a model based on the evolutionary game framework. This may be the cost of a deterministic dynamic model stemming from the evolutionary game framework. On the whole, when the advantages of the evolutionary game framework are taken into consideration, the cost is offset.

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